# IMPLICIT INCORPORATION OF NONLINEAR ELEMENTS FOR UNCONDITIONALLY STABLE FDTD ANALYSIS AT COARSE TIME-STEPS

Malgorzata Celuch - Marcysiak, Wojciech K.Gwarek

Institute of Radioelectronics, Warsaw University of Technology,
Nowowiejska 15/19, 00-665 Warsaw, Poland.

## ABSTRACT

A hybrid 3D FDTD algorithm with a fully implicit interface to lumped nonlinear multiport devices is presented. The algorithm is unconditionally stable and converges for arbitrarily strong nonlinearities, high voltages and with any time-step satisfying the Courant condition. In case of a strongly nonlinear bipolar transistor, the new algorithm is by over two orders of magnitude faster than previous FDTD schemes with explicit or semi-implicit interfaces.

## INTRODUCTION

A prospect to enhance accurate design of nonlinear high-speed MMICs resides in conducting a unified electromagnetic simulation of the whole circuit, directly including nonlinearities. Although in principle the FDTD or TLM analysis can be based on microscopic-scale discretization, presently available computers are still by several orders of magnitude too slow to realize this task. For example, electromagnetic analysis of the active area of a DCFL inverter presented in [1] required a 3D model of about 60 thousand cells, and 300 thousand iterations to monitor a 500ps pulse. Our estimation of the computer effort for this problem is about 300 hours on an IBM PC 486. Extending space discretization to the entire circuit would lead to prohibitive computing time even on supercomputers.

We can resort to hybrid modelling which combines electromagnetic simulation of the linear subcircuit with lumped models of semiconductor diodes and transistors. We have previously developed a fully implicit interface for incorporating a diode into a 2D FDTD grid [2][3]. Other authors have followed a different approach, using explicit [4] or semi-implicit [5]..[9] interfacing in either FDTD [4]..[7] or TLM [8][9]. The explicit or semi-implicit interfaces are simpler and provide a single-step solution of the nonlinear equations at each time-step. However, they diverge for problems involving strong nonlinearities and large driving signals.

We will show that under such circumstances, the fully implicit interface proves computationally more effective, and we develop such an interface for the most general case of a lumped nonlinear multiport incorporated in a 3D FDTD grid.

TH 2B

## INTERFACING LUMPED ELEMENTS

From the point of view of the linear circuit, each nonlinear port can be represented by a nonlinear admittance and a current source controlled by voltages at other ports so that:

$$ {}_0I_P = {}_0I_D({}_0U_z) + {}_0I_C({}_1U_z, {}_2U_z, \dots, {}_{n-1}U_z) \quad (1) $$

where prescript 0 denotes the considered port while 1...$n$-1 - the remaining ports of the nonlinear component. Let us assume that the nonlinear port is coupled to the $E_z$ field in a particular FDTD cell. FDTD equations for updating the $E_z$ field result from applying the Maxwell equation:

$$ \iint \left[ \frac{\partial \bar{D}}{\partial t} + \bar{J} \right] d\bar{S} = \oint \bar{H}\, d\bar{c} \quad (2) $$

over surface $S$ perpendicular to the $z$-axis. In the presence of the nonlinear port (and assuming lossless medium for clarity of description) we have:

$$ \iint \bar{J}\, d\bar{S} = \bar{i}_z I_P \quad (3) $$

With central finite differences we get:

$$ {}_0U_z^{k+1} = {}_0U_z^k + \frac{\Delta t}{\frac{\Delta x \Delta y}{\Delta z}\varepsilon} \left[ \begin{array}{l} {}_{\frac{\Delta x}{2}}K_y^{k+\frac{1}{2}} - {}_{-\frac{\Delta x}{2}}K_y^{k+\frac{1}{2}} - {}_{\frac{\Delta y}{2}}K_x^{k+\frac{1}{2}} \\ + {}_{-\frac{\Delta y}{2}}K_x^{k+\frac{1}{2}} - {}_0I_P^{k+\frac{1}{2}} \end{array} \right] \quad (4) $$

The electric and magnetic potentials $U_z$=$E_z\Delta z$, $K_x$=$H_x\Delta x$, $K_y$=$H_y\Delta y$ simplify the interfacing problem. At each iteration a nonlinear system of 2$n$ interface equations must be solved ($n$ voltage equations of the form (4) and $n$ current equations of the form (1)). Three approaches can be distinguished:

### a) Explicit [4]:

At each time step the nonlinear port is modelled by its current calculated at a previous time-step:

$$ {}_0I_P^{\,k+0.5} = {}_0I_P^{\,k} $$

This is inconsistent with the leap-frog discretization of the Maxwell equations in FDTD. Explicit interface leads to instabilities even for weak nonlinearities [4][5].

### b) Semi-implicit [1][5][6][8][9]:

At each time-step the nonlinear element is modelled by its differential parameters corresponding to the previous time step. Generalizing the algorithms of [1][5][6][8][9] to multiport nonlinearities, we can rewrite (4) as:

$$ {}_iU_z^{k+1} = {}_i\tilde{U}_z^{k+1} - {}_iR I_P^{k+1} \quad (i = 1,\dots,n) \quad (5) $$

where

$$ {}_i\tilde{U}_z^{k+1} = {}_iU_z^k + \frac{\Delta t}{\frac{\Delta x \Delta y}{\Delta z}\varepsilon} \left[ \begin{array}{l} {}_{\frac{\Delta x}{2}}K_y^{k+\frac{1}{2}} - {}_{-\frac{\Delta x}{2}}K_y^{k+\frac{1}{2}} - {}_{\frac{\Delta y}{2}}K_x^{k+\frac{1}{2}} + \\ {}_{-\frac{\Delta y}{2}}K_x^{k+\frac{1}{2}} - \frac{1}{2}\, {}_iI_P^k \end{array} \right] \quad (6) $$

$$ {}_iR = \frac{1}{2} \frac{\Delta t}{\frac{\Delta x \Delta y}{\Delta z}\varepsilon} \quad (7) $$

We rewrite (1) as:

$$ {}_iI_P^{k+1} = {}_i\tilde{I}_P^{k+1} + \sum_{j=1}^{n} {}_{ji}g^k \cdot {}_jU_z^{k+1} \quad (i=1,\dots,n) \quad (8) $$

$$ {}_i\tilde{I}_P^{k+1} = {}_iI_P^k - \sum_{j=1}^{n} {}_{ji}g^k \cdot {}_jU_z^k \quad (i=1,\dots,n) \quad (9) $$

Semi-implicit procedures remain stable only for sufficiently small time steps $\Delta t<\Delta t_{max}$, $\Delta t_{max}$ being related to the slope of the nonlinear characteristics [10]. This explains why the value of $\Delta t$ much below the typical FDTD discretization has been used in [1][5][6].

### c) Fully implicit:

The nonlinear element is represented by its differential conductances and transconductances corresponding to the current time-step, modifying (8) to (10):

$$ {}_iI_p^{k+1} = {}_iI_p^{k} + \sum_{j=1}^{n} {}_{ji}g^{k+1} \cdot \left( {}_jU_z^{k+1} - {}_jU_z^{k} \right) \quad (10) $$

The fully implicit interface guarantees stability for arbitrarily strong nonlinearities and at any discretization. For fast convergence, we have developed a specialized scheme of nonlinear integration which we call a monotone admittance method. Its key features have been outlined in [3].

## EXAMPLES

We consider a strip-line circuit incorporating a transistor as in Fig.1. This is a 1D problem for which a benchmarking solution can be obtained from SPICE.

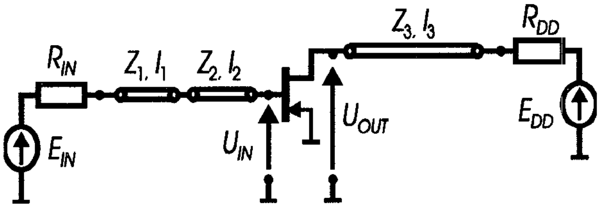


Fig.1. A test circuit for the FDTD method incorporating nonlinearities.

**Example 1:** GaAsFET inverter

We have simulated a GaAsFET inverter of Fig.1. A majority of transistor parameters have been taken from [1], and SPICE default values used for parameters unspecified in [1]. As expected for a circuit involving only 1D wave propagation, the agreement of FDTD with SPICE is very good (Fig.2). Yet in contrast to SPICE, our method is directly applicable to more complicated arbitrarily shaped 3D microwave circuits.

As a consequence of lumped element representation of the GaAsFET in FDTD, and of using an implicit interface, the results of Fig.2 have been obtained with 10 iterations per pulse rise time, in place of 24000 iterations per pulse rise time needed in [1] for the same transistor discretized on a microscopic scale.

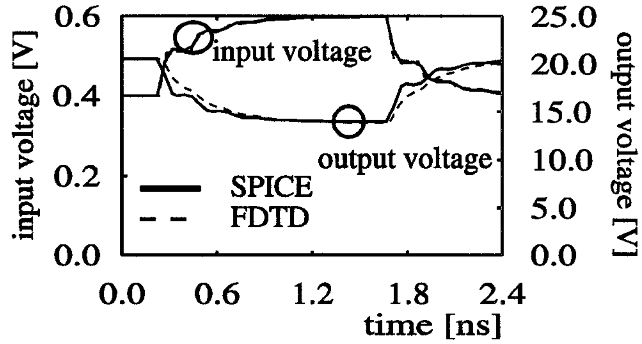


Fig.2. Input ($U_{IN}$) and output ($U_{OUT}$) voltage of a GaAsFET inverter.
$\beta_0$=0.0287 A/V$^2$,$C_{GS0}$=$C_{GD0}$=9.59fF,
$\lambda$=0.1V$^{-1}$, $\alpha$=2 V$^{-1}$, $V_{T0}$=0.21V
$R_S$=$R_D$=50Ω $E_{DD}$=24 V, $R_{IN}$=$Z_1$=50 Ω,
$Z_2$=20 Ω, $l_1$=$l_2$=30mm, $Z_3$=$R_{DD}$=10 kΩ,
driving pulse $E_{IN}$ : duration 1.4 ns, rise and fall time 40 ps, low and high level 0.4 V, 0.6 V

**Example 2:** fully implicit versus semi-implicit interface to a bipolar transistor

We insert a bipolar transistor with B-E port current described by:

$$ I_D = I_S \exp\left[ \frac{kT}{q} U_z - 1 \right] \quad (11) $$

into a circuit of Fig.1 with $E_{IN}(t)$=$E_0\sin(2\pi f_0 t)$, $f_0$=20GHz, $E_0$=1V, $l_1$+$l_2$=0.5$c/f_0$, $Z_1$=$Z_2$=1Ω, $R_{IN}$=1Ω. We assume $I_s$=100pA.

In Fig.3 we show voltage waveforms obtained at the B-E port with various time steps using a semi-implicit and a fully implicit interface. At fine discretization both behave analogously. However, at coarser discretization numerical noise contaminates the semi-implicit solution (Fig.3b), and at $T_0/\Delta t$=15 it leads

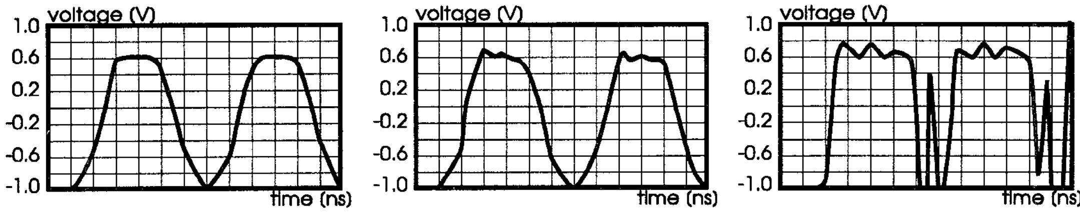


Fig.3. Voltage waveforms obtained at the input (B-E) of a bipolar transistor: a) semi-implicit interface and fully implicit $T/\Delta t$=15, b) semi-implicit $T/\Delta t$=22.5, c) semi-implicit $T/\Delta t$=15.

to instability (Fig.3c). If the driving voltage amplitude is increased to $E_0$=10V, the discretization of $T_0/\Delta t$=675 becomes necessary for stability of the semi-implicit algorithm. On the other hand, we have checked that the fully implicit algorithm developed in this work remains stable with the discretization of $T_0/\Delta t$=15 with $E_0$=10V.

## CONCLUSIONS

We have presented a novel FDTD algorithm with a fully implicit interface to the lumped nonlinear multiport device. Efficiency of hybrid modelling combined with implicit interfacing has been demonstrated for a GaAsFET inverter and a bipolar transistor circuit.

## REFERENCES

[1] R.H.Voelker, R.J.Lomax, "A finite-difference transmission line matrix method incorporating a nonlinear device model", *IEEE Trans. Microwave Theory Tech.*, MTT-38, No.3, March 1990, pp.302-312.

[2] M.Celuch, "*A method of analysis of planar circuits incorporating nonlinear elements*", M.Sc. thesis, Warsaw Univ. of Technology, Warsaw 1988

[3] M.Celuch-Marcysiak, W.K.Gwarek, "Unconditionally stable time-domain electromagnetic simulation of nonlinear microwave devices", *Proc. of Microwave and Optronics Conf.- MIOP*, Stuttgart 1993, pp.579-583.

[4]. W.Sui, D.A.Christensen, C.H.Durney, "Extending the two-dimensional FDTD method to hybrid electromagnetic systems with active and passive lumped elements", *IEEE Trans. Microwave Theory Tech.*, MTT-40, No.4, April 1992, pp.724-730.

[5] M.Piket-May, A.Taflove, J.Baron, "FD-TD modeling of digital signal propagation in 3-D circuits with passive and active loads", *IEEE Trans. Microwave Theory Tech.*, MTT-42, No.8, August 1994, pp.1514-1523..

[6] P.Ciampolini, P.Mezzanotte, L.Roselli, D.Sereni, R.Sorrentino, P.Torti, "Simulation of HF circuits with FDTD technique including non-ideal lumped elements", *IEEE .MTT-S Digest*, San Diego 1995, pp.361-364.

[7] V.A.Thomas, M.E.Jones, M.Piket-May, A.Taflove, E.Harrigan, "The use of SPICE lumped circuit as sub-grid models for FDTD analysis", *IEEE Microwave and Guided Wave Letters*, vol.4, No.5, may 1994, pp.141-143.

[8]. W.J.R.Hoefer, B.Isele, P.Russer, "Modelling of nonlinear active devices in TLM", *IEE Intl. Conf. 'Computation in Electromagnetics'*, London 1991, pp.327-330.

[9] M.I.Sobhy, E.A.Hosny, M.H.Abd Eli-Azeem, K.W.Royer, "Simulation of nonlinear and anisotropic structures in 3D TLM", *European Microwave Conf. Proc.*, Bologna 1995, pp.830-833.

[10] L.O.Chua,P.M.Lin, "*Computer-aided analysis of electronic circuits*", Prentice-Hall, Inc., New Jersey, USA, 1975.